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# A critical approach to the concept of a polar, low-altitude LARES satellite

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## Abstract

According to very recent developments of the LARES mission, which would be devoted to the measurement of the general relativistic Lense–Thirring effect in the gravitational field of the Earth with Satellite Laser Ranging, it seems that the LARES satellite might be finally launched in a polar, low-altitude orbit by means of a relatively low-cost rocket. The observable would be the node only. The Lense–Thirring effect on it would consist of a secular linear trend. The biasing classical secular nodal precessions due to the even zonal harmonics of the geopotential, which represent the major source of uncertainty, exactly vanished if and only if the orbit would be exactly polar. Due to the small altitude, even small possible deviations from the projected inclination, which might be induced by the orbital injection errors, would yield a rather large systematic error due to the even zonal harmonics of geopotential in the measurement of the relativistic nodal shift. So, in this paper we show how such a configuration, in fact, to the present level of knowledge of the terrestrial gravitational field according to the EGM96 gravity model, should be of relatively little utility in increasing the obtainable accuracy in measuring the Lense–Thirring effect with respect not only to the originally proposed supplementary LARES–LAGEOS configuration, but also to the present LAGEOS–LAGEOS II experiment which has a total accuracy of the order of 20% – 30%. Maybe the situation might be improved, at least to a certain extent, when the new, more accurate Earth gravity models from the CHAMP and GRACE missions will be available.

# 1 Introduction

In its weak-field and slow-motion approximation General Relativity predicts that, among other things, the orbit of a test particle freely falling in the gravitational field of a central rotating body is affected by the so called gravitomagnetic dragging of the inertial frames or Lense-Thirring effect. More precisely, the longitude of the ascending node  $\Omega$  and the argument of the perigee  $\omega$  of the orbit undergo tiny precessions according to [Lense and Thirring, 1918]

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (1)$$

$$\dot{\omega}_{\text{LT}} = -\frac{6GJ \cos i}{c^2 a^3 (1 - e^2)^{\frac{3}{2}}}, \quad (2)$$

in which  $G$  is the Newtonian gravitational constant,  $J$  is the proper angular momentum of the central body,  $c$  is the speed of light *in vacuum*,  $a$ ,  $e$  and  $i$  are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test particle. The Lense-Thirring precessions for the LAGEOS satellites amount to

$$\dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} = 31 \text{ mas/y}, \quad (3)$$

$$\dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} = 31.5 \text{ mas/y}, \quad (4)$$

$$\dot{\omega}_{\text{LT}}^{\text{LAGEOS}} = 31.6 \text{ mas/y}, \quad (5)$$

$$\dot{\omega}_{\text{LT}}^{\text{LAGEOS II}} = -57 \text{ mas/y}. \quad (6)$$

The first measurement of this effect in the gravitational field of the Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing passive geodetic satellites LAGEOS and LAGEOS II [Ciufolini *et al.*, 1998]. The observable [Ciufolini, 1996] is a linear trend with a slope of 60.2 milliarcseconds per year (mas/y in the following) and includes the residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II<sup>1</sup>. The total relative accuracy of the measurement of the solve-for parameter  $\mu_{\text{LT}}$ , introduced in order to account for this general relativistic effect, is of the order of  $2 \times 10^{-1}$  [Ciufolini *et al.*, 1998].

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<sup>1</sup>The perigee of LAGEOS was not used because it introduces large observational errors due to the smallness of the LAGEOS eccentricity [Ciufolini, 1996] which amounts to 0.0045.

In this kind of satellite-based space experiments the major source of systematic errors is represented by the aliasing trends due to the classical secular precessions [Kaula, 1966] of the node and the perigee induced by the mismodelled even zonal harmonics of the geopotential  $J_2$ ,  $J_4$ ,  $J_6$ , ... Indeed, according to the present knowledge of the Earth's gravity field based on the EGM96 model [Lemoine *et al.*, 1998], they amount to a large part of the gravitomagnetic precessions of interest, especially for the first two even zonal harmonics. In the performed LAGEOS-LAGEOS II Lense-Thirring experiment the adopted observable allows for the cancellation of the static and dynamical effects of  $J_2$  and  $J_4$ . The remaining higher degree even zonal harmonics affects the measurement at a 13% level.

In order to achieve a few percent accuracy, in [Ciufolini, 1986] it was proposed to launch a passive geodetic laser-ranged satellite- the former LAGEOS III - with the same orbital parameters of LAGEOS apart from its inclination which should be supplementary to that of LAGEOS.

This orbital configuration would be able to cancel out exactly the classical nodal precessions, which are proportional to  $\cos i$ , provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LAGEOS III and LAGEOS

$$\delta\dot{\Omega}^{\text{III}} + \delta\dot{\Omega}^{\text{I}} = 62\mu_{\text{LT}}. \quad (7)$$

Later on the concept of the mission slightly changed. The area-to-mass ratio of LAGEOS III was reduced in order to make less relevant the impact of the non-gravitational perturbations and the eccentricity was enhanced in order to be able to perform other general relativistic tests: the LARES was born [Ciufolini, 1998]. The orbital parameters of LAGEOS, LAGEOS II and LARES are in Table 1.

Table 1: Orbital parameters of LAGEOS, LAGEOS II LARES and POLARES.

Orbital parameter	LAGEOS	LAGEOS II	LARES	POLARES
$a$ (km)	12,270	12,163	12,270	8,378
$e$	0.0045	0.014	0.04	0.04
$i$ (deg)	110	52.65	70	90

Recent developments of the concept of the twin satellites in supplementary orbits have led

to the discover of new, possible gravitomagnetic observables based on the use of the perigees as well [Iorio, 2002a] and of unexpected connections with the gravitomagnetic clock effect [Iorio and Lichtenegger, 2002].

Unfortunately, at present we do not know if the LARES mission will be approved by any space agency. Although much cheaper than other proposed and/or approved space-based missions, funding is the major obstacle in implementing the LARES project. The most expensive part is the launching segment.

Very recently the possibility of launching the LARES satellite into an orbit with  $a = 8,378$  km,  $i = 90$  deg,  $e = 0.04$  and using as observable its node has been considered [Lucchesi and Paolozzi, 2001]. In the following we will name POLARES the LARES satellite in such proposed polar orbit. The Lense–Thirring secular rate of the POLARES node would amount to 96.9 mas/y. Then, the observable would be

$$\delta\dot{\Omega}^{\text{PL}} = 96.9\mu_{\text{LT}}, \quad (8)$$

i.e. a linear secular trend with a slope of 96.9 mas/y. The choice of a so low altitude is motivated by the need of using a cheap rocket launcher; so, it must certainly be considered as an admirable and important further effort towards the practical realization of the LARES project. The polar orbit would allow to prevent the aliasing effects of the mismodelled classical secular precessions of the node induced by the even  $l = 2n$  zonal  $m = 0$  coefficients of the multipolar expansion of the static part of the geopotential. The non-gravitational perturbations [Lucchesi, 2001; 2002; Lucchesi and Paolozzi, 2001] would represent a minor problem.

In this paper we wish to critically analyze such important and interesting evolution of the LARES concept.

## 2 The POLARES

### 2.1 The node-only scenario

Here we will show that the proposed POLARES configuration and the use of the node only as gravitomagnetic observable would present some drawbacks due to the gravitational static and time-varying perturbations.

In regard to the classical secular precessions induced by the centrifugal oblateness of the Earth, it should be considered that the choice of the inclination would be of crucial importance. Such orbital element is affected neither by the secular perturbations induced by the even zonal harmonics of the geopotential [Kaula, 1966] nor by the semisecular 18.6-year and 9.3-year tidal perturbations [Iorio, 2001]. On the other hand, due to possible orbital injection errors which are closely related to the quality of the rocket launcher to be used, the POLARES inclination would be far from being exactly 90 deg. The relatively low altitude of POLARES would enhance the impact of even small departures of  $i_{PL}$  from its projected nominal value on the systematic error due to the classical static even zonal harmonics of geopotential  $\delta\mu_{LT}^{\text{even zonals}}$ . This is clearly shown in Figure 1 in which it has been calculated by means of the covariance matrix of the EGM96 Earth gravity model up to degree  $l = 20$ . It should be considered that, contrary to the

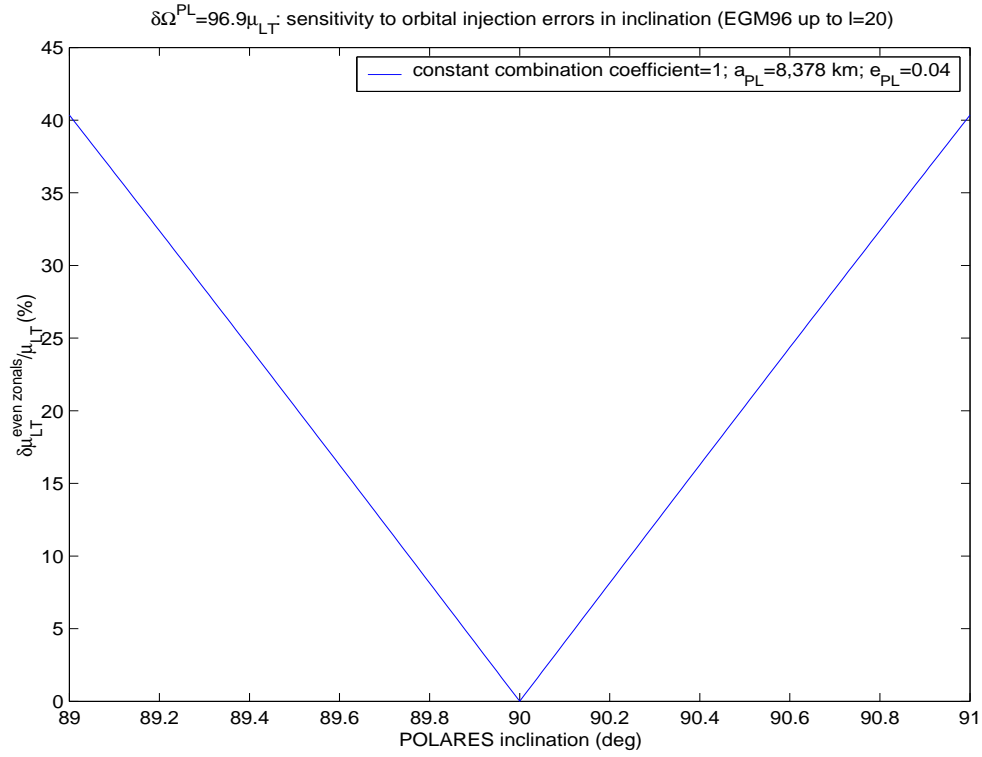


Figure 1: POLARES scenario: influence of the orbital injection errors in the POLARES inclination on the zonal error in the Lense–Thirring nodal shift.

LAGEOS satellites which are almost insensitive to the even zonal harmonics of degree higher than  $l = 20$ , this would not be the case for the POLARES, due to its projected altitude of

2,000 km. Then, it might turn out that the estimates of Figure 1 are optimistic.

If we consider that the impact of the mismodelled even zonal harmonics of the geopotential on the current LAGEOS–LAGEOS II Lense–Thirring experiment is of the order of 13%, according to EGM96, a certain weakness of the POLARES node–only scenario becomes apparent. In regard to the possibilities offered by the forthcoming new gravity models from the CHAMP and GRACE missions, it should be considered that the major improvements in the accuracy of the even zonal coefficients of the geopotential are expected for the first degrees. Now, in the LAGEOS–LAGEOS II Lense–Thirring experiment the first two even zonal harmonics are cancelled out and the terms of degree higher than  $l = 20$  are not relevant because the LAGEOS and LAGEOS II orbits are not sensitive to them. So, the systematic error due to the geopotential, in this case, is due to the even zonal harmonics from  $l = 6$  to  $l = 20$ . Then, the CHAMP and GRACE missions would certainly improve such error. On the other hand, the POLARES configuration would be sensitive both to the first two even zonal harmonics, which should be greatly improved by the CHAMP and GRACE results, and to the terms of degree higher than  $l = 20$ , for which the improvements should be less relevant. So, even in this perspective, maybe that a simple reanalysis of the LAGEOS and LAGEOS II data should be more fruitful than the node–only POLARES choice.

Such conclusion is enforced also by a simple evaluation of the impact of some tidal perturbations<sup>2</sup> [Iorio, 2001]. Putting aside the fact that the 18.6-year lunar tide, which has a period of 18.6 years due to the lunar node only motion and a large amplitude, is a  $l = 2$ ,  $m = 0$  constituent which would affect the POLARES node for  $i_{PL} \neq 90$  deg, let us draw our attention to the  $l = 2$ ,  $m = 1$   $K_1$  tesseral tide. It is an important constituent which exerts a relevant perturbing action on the node of a satellite<sup>3</sup> and has the same period of just the node of the satellite. It turns out that for, say,  $i_{PL} = 89.8$  deg the period of the POLARES node would amount to 26,801 days, i.e. 73.3 years. This means that, even by not considering at all the even zonal perturbations, the mismodelled part of the  $K_1$  tide itself would resemble a superimposed aliasing trend over a reasonable observational time span of a few years and would completely

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<sup>2</sup>The amplitude of the tidal perturbations on the node is proportional to  $(1/\sin i)dF_{lmp}/di$  where  $F_{lmp}(i)$  are the inclination functions [Kaula, 1966]. For the even zonal ( $m = 0$ ), tesseral ( $m = 1$ ) and sectorial ( $m = 2$ ) terms they are  $F_{201} = (3/4)\sin^2 i - (1/2)$ ,  $F_{211} = -(3/2)\sin i \cos i$  and  $F_{221} = (3/2)\sin^2 i$ , respectively.

<sup>3</sup>For LAGEOS it has a nominal amplitude of the order of  $10^3$  mas [Iorio, 2001]

mask the Lense–Thirring trend.

## 2.2 The combined residuals scenario

Let us try to see what could happen by inserting the node and the perigee of POLARES in a suitable combination of orbital residuals with the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II along the lines sketched in [Ciufolini, 1996; Iorio, 2002b]. Recall that if  $N$  orbital elements are present in such combinations, the effects of the first  $N - 1$  even zonal harmonics of the geopotential are cancelled out, irrespectively of the orbital geometry of the employed satellites.

It turns out that by combining the nodes of LAGEOS, LAGEOS II and POLARES and the perigees of LAGEOS II and POLARES in

$$\delta\dot{\Omega}^I + c_1\delta\dot{\Omega}^{II} + c_2\delta\dot{\Omega}^{PL} + c_3\delta\dot{\omega}^{II} + c_4\delta\dot{\omega}^{PL} = X_{LT}\mu_{LT} \quad (9)$$

the inclination  $i_{PL} = 90$  deg is singular in the sense that the slope of the relativistic effect diverges because the coefficient  $c_2$  with which the node of POLARES enters the combination diverges. For values of  $i_{PL}$  close to 90 deg Figure 2 and Figure 3 show that the systematic error due to the remaining  $\delta J_{10}, \delta J_{12}, \dots$ , according to the covariance matrix of EGM96 up to degree  $l = 20$ , would seriously affect the measurement of the Lense–Thirring linear trend. Moreover, the impact of all the gravitational and non–gravitational time–varying perturbations would be greatly enhanced by the large values of  $c_2$ . The situation would not be better even if we would include only the perigee of POLARES in the combined residuals

$$\delta\dot{\Omega}^I + c_1\delta\dot{\Omega}^{II} + c_2\delta\dot{\omega}^{II} + c_3\delta\dot{\omega}^{PL} = X_{LT}\mu_{LT}, \quad (10)$$

as shown by Figure 4.

## 3 Conclusions

In this paper the proposal of putting the LARES satellite into a polar, elliptical orbit with an altitude of 2,000 km in order to look at its gravitomagnetic secular node shift has been critically analyzed.

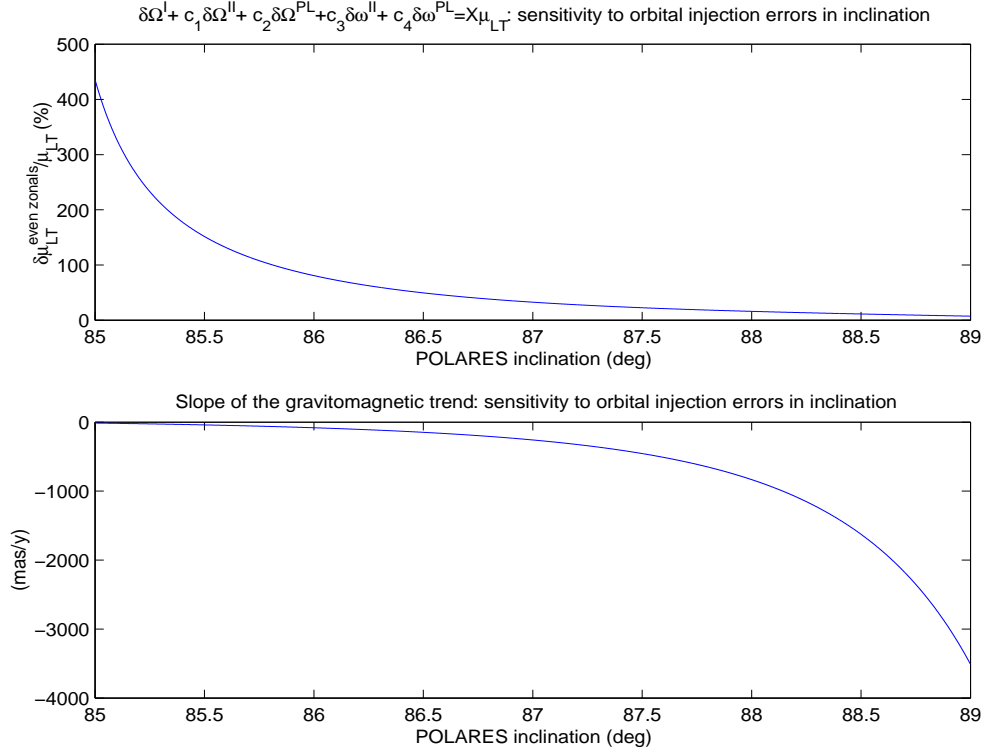


Figure 2: POLARES, LAGEOS, LAGEOS II scenario: influence of the orbital injection errors in the POLARES inclination on the zonal error in the combined residuals with the node of POLARES.

The key point is that the mismodelled classical nodal secular precessions induced by the even zonal coefficients of the multipolar expansion of the terrestrial gravitational field vanished if and only if the inclination of the satellite would be exactly 90 deg. Of course, mainly due to possible orbital injection errors, this could never happen. It turns out that the low altitude of the proposed orbital configuration, and the consequent high sensitivity to the higher even degree zonal terms of the geopotential, would greatly enhance the impact of even small departures of the real values of the inclination from the nominal value of 90 deg. For example, for just  $i_{PL} = 90 \pm 0.2$  deg the systematic gravitational error would be of the order of 5%–10%, according to the EGM96 Earth gravity model up to  $l = 20$ , which is of the same order of magnitude of the present LAGEOS–LAGEOS II experiment (Its total error, including various systematic gravitational and non-gravitational perturbations, is of the order of 20%–30%). Of course, such a situation would be further made critical by the possible use of a low-cost launcher which, unavoidably, would induce not negligible orbital injection errors. Moreover, the tesseral



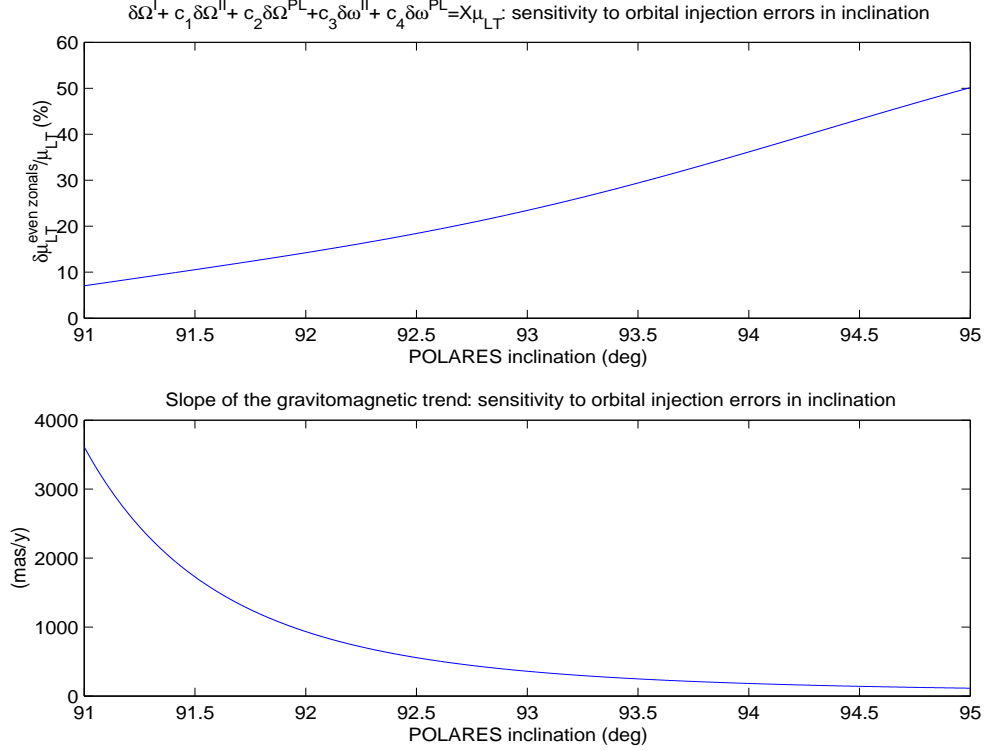


Figure 3: POLARES, LAGEOS, LAGEOS II scenario: influence of the orbital injection errors in the POLARES inclination on the zonal error in the combined residuals with the node of POLARES.

$K_1$  tidal perturbation, which has the same period of the satellite's node, would induce a secular aliasing trend over an observational time span of a few years because its period would amount to several tens of years for near polar orbits.

Another important point is that the proposed POLARES would not yield substantial improvements also in the context of the combined residuals approach which allows to cancel out the contribution of the first even zonal coefficients of the geopotential irrespectively of the inclination of the satellites. Indeed, it turns out that a combination including the nodes of LAGEOS, LAGEOS II, POLARES and the perigees of LAGEOS II and POLARES would be not defined for  $i_{PL} = 90$  deg because the coefficient weighing the node of the LARES would go to infinity. For very small deviations of  $i_{PL}$  from such a critical value the systematic error induced by the remaining even zonal harmonics would amount to 10%–20%. Moreover, the coefficient with which the node of POLARES would enter the combination would be much more larger than unity and would greatly enhance the impact of the gravitational and non-gravitational

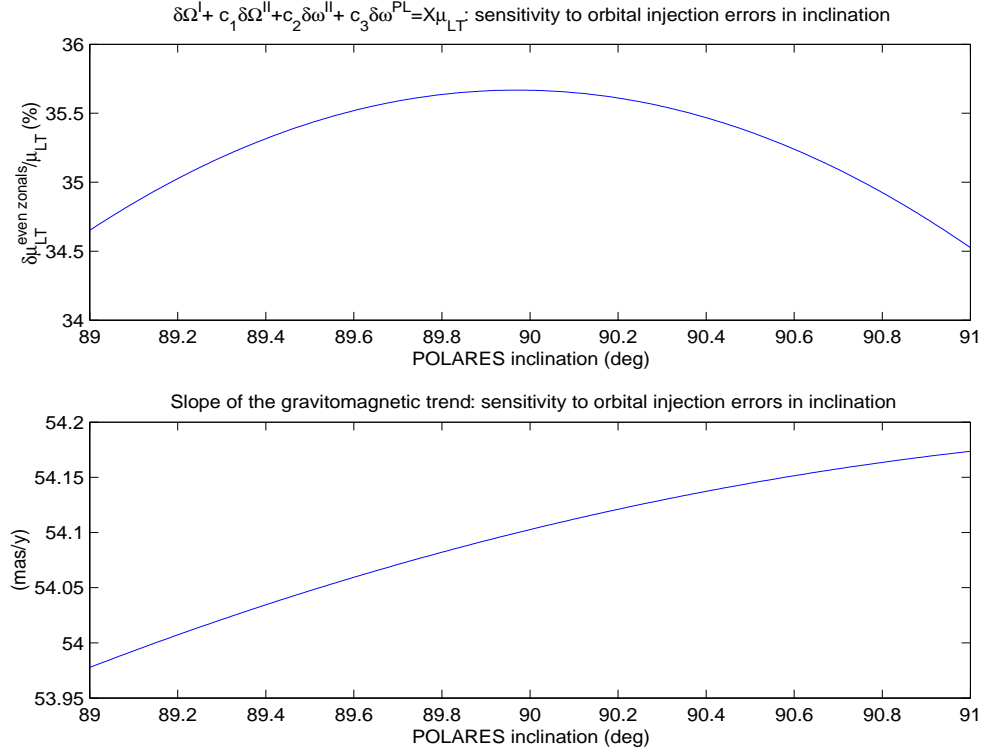


Figure 4: POLARES, LAGEOS, LAGEOS II scenario: influence of the orbital injection errors in the POLARES inclination on the zonal error in the combined residuals without the node of POLARES.

time-dependent perturbations. Even the use of the nodes of LAGEOS and LAGEOS II and the perigees of LAGEOS II and POLARES, in which case  $i_{PL} = 90$  deg would not create problems, would not yield benefits because the systematic gravitational error would be of the order of 20%–30%.

Moreover, the accuracy of the practical data reduction from such version of LARES would be affected by the atmospheric drag.

Finally, we can conclude that the proposed low-cost version of the LARES mission would not yield any significant improvements in the measurement of the elusive Lense–Thirring effect, according to the present-day level of knowledge of the Earth’s gravitational field. Perhaps, the situation could improve to a certain extent with the new, more accurate gravity models from CHAMP and GRACE missions which should become available in the next few years. On the contrary, the original concept of the couple of supplementary satellites would deserve grater attention thanks to its much richer spectrum of high accuracy relativistic observables.

## References

- [Ciufolini, 1986] Ciufolini, I., Measurement of Lense–Thirring drag on high-altitude laser ranged artificial satellite, *Phys. Rev. Lett.*, *56*, 278-281, 1986.
- [Ciufolini, 1996] Ciufolini, I., On a new method to measure the gravitomagnetic field using two orbiting satellites, *Il Nuovo Cimento*, *109A*, *12*, 1709-1720, 1996.
- [Ciufolini, 1998] Ciufolini, I., The Concept of the LARES experiment, in *LARES Phase–A Study*, pp. 16-33, Rome, 1998.
- [Ciufolini et al., 1998] Ciufolini, I., E. Pavlis, F. Chieppa, E. Fernandes-Vieira, and J. Pérez-Mercader, Test of General Relativity and Measurement of the Lense-Thirring Effect with Two Earth Satellites, *Science*, *279*, 2100-2103, 1998.
- [Iorio, 2001] Iorio, L., Earth tides and Lense-Thirring effect, *Celest. Mech.*, *79*(3), 201-230, 2001.
- [Iorio, 2002a] Iorio, L., On a new observable for measuring the Lense-Thirring effect with Satellite Laser Ranging, gr-qc/0206074, 2002a.
- [Iorio, 2002b] Iorio, L., Testing General Relativity with LAGEOS, LAGEOS II and Ajisai laser–ranged satellites, *J. of the Geodetic Soc. of Japan*, *48*(1), 13–20, 2002b.
- [Iorio, 2002] Iorio, L., and H.I.M. Lichtenegger, paper in preparation, 2002.
- [Kaula, 1966] Kaula, W. M., *Theory of Satellite Geodesy*, 124 pp., Blaisdell Publishing Company, Waltham, 1966
- [Lemoine et al., 1998] Lemoine, F. G., et al., The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, 1998.
- [Lense and Thirring, 1918] Lense, J., and H. Thirring, Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, *Phys. Z.*, *19*, 156-163, 1918, translated by Mashhoon, B., F.

W. Hehl, and D. S. Theiss, On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, *Gen. Rel. Grav.*, *16*, 711-750, 1984.

[*Lucchesi*, 2001] Lucchesi, D., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part I, *Plan. and Space Sci.*, *49*, 447-463, 2001.

[*Lucchesi*, 2002] Lucchesi, D., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part II, to be published in *Plan. and Space Sci.*, 2002.

[*Lucchesi and Paolozzi*, 2001] Lucchesi, D., and A. Paolozzi, A cost effective approach for LARES satellite, paper presented at *XVI Congresso Nazionale AIDAA*, *24-28 September 2001, Palermo*, 2001.